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STABILITY PROPERTIES OF QUANTUM IMPULSIVE SOLOW GROWTH MODEL

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Abstract

We generalize impulsive Solow growth model to quantum calculus of Hudsonand Parthasarathy formulation, we investigate boundedness and stability of solution of economic model that are subject to instantaneous perturbationsdue to endogenous delays. We apply the direct Lyapunov method in establishing sufficient conditions for uniform, ultimate boundedness and uniform, asymptoticstability of thes olutions.

AMS Subject Classification: 81S25, 34A37, 34D10.
Keyword: Direct Lyapunov method, Uniform asymptotic stability, Uniform ultimate boundedness, Quantum Impulsive Solow Growth Model.

INTRODUCTION

In the recent years, the field of quantum economics & finance have appeared in other to interpret erratic stock market behavior with the use of quantum mechanical concepts [1, 4, 5, 11, 15, 17, 18, 20]. Segal & Segal introduced quantum effects into the Merton-Black-Scholes model in other to incorporate market features such as the impossibility of simultaneous measurement of prices and their instantaneous derivatives. They did that by adding to the Brownian motion$B_t$used to represent the evolution of public information affecting the market, a process$Y_t$which represents the influence of factors not simultaneously measurable with those involvedB_t. They sketch a calculus for dealing with such processes. They concluded that the combined processes$aB_t + bY_t$may be represented as (in their notation) $\Phi((a+ib)\chi_{0,t})$where for a Hilbert space element $f, e^{i\Phi(f)}$ is the characteristic function of the interval [0, t]. In the context of the Hudson and Parthasarathy quantum calculus [10, 14], simple linear combinations of $\Phi(f)$and $\Phi(if)$defined the Boson Fock space annihilator and creator operatorsA_f *and* A_f^+. Segal and Segal used $\Phi(\chi_{0,t})$ as the basic integrator process with integrands restricted to a special class of exponential processes. In view of the above reduction of Φ to A and A^+, it is logical to study economic growth using as integrators the annihilator and creator processes of Hudson-Parthasarathy quantum stochastic calculus, thus exploiting its much larger class of integrable processes than the one considered in [18]. The Hudson-Parthasarathy calculus has a wide range of applications. For applications examples on control Theory we refer to [2] and [9] and the references therein.

Fundamental Structures

Following the fundamental structures in [3,10,12,18], we employ the locally convex space $\tilde{A}$of a non-commutativeprocess whose topology is generated by family of seminorms,

$||x||_{\eta,\xi} = |\langle\eta, x\xi\rangle$. The underlying element of $\tilde{A}$ consist linear maps from$D \underline{\otimes} E$ into $R\otimes\Gamma(L^2_\gamma(R_+))$ having domain of their adjoints containing . $D \underline{\otimes} E$

Definitions

Let D and E be two Hilbert spaces. The algebraic tensor product of D and E denoted by $D \underline{\otimes} E$ is the scalar product induced by the sesquilinear forms of D and E. It has natural positive definite sesquilinear form and its completion is a Hilbert space. The completion of the algebraic tensor product in these two norms are called the projective and injective tensor products. The topologies of the locally convex spaces are given by families of seminorms.

Let $\eta = c\otimes e(\alpha)$, $\xi = d\otimes e(\beta)$be exponential vectors in$D \underline{\otimes} E$ such that

i. $P : I \rightarrow P_{wac}(\tilde{A})$, $P(t,x)(\eta,\xi) = \langle\eta, P_{\alpha,\beta}(t,x)\xi\rangle$

Let $P_{\alpha,\beta} =$ P, P_{wac} is the space of the weakly absolutely continuous process in $\tilde{A}$

ii. The space$Pad(I,\tilde{A})_{wac} = \{X : \tilde{A}\,;\ X\ is\ adapted\ absolutely\ continuous$

$$everywhereexcept\ for\ some\ t_k\ at\ which\ X(t_k^-) and\ X(t_k^+),\ k = 1,2,\dots,m\ exists\ and\ X(t_k^-) = X(t_k^+)\}.$$

iii. For each pair$\eta,\xi \in D \underline{\otimes} E$we define the space of complex valued numbers associated with (i) as $Pad(I,\tilde{A})_{wac,\eta\xi,} = \{\langle\eta, \Phi(.)\xi\rangle : \Phi Pad(I,\tilde{A})_{wac}\}$

iv. On $Pad(I,\tilde{A})_{wac}$ we define a seminorm$\left||\Phi|\right|_{p,\eta\xi} = \sup\{\left||\Phi|\right|_{\eta\xi}, t \in I\}$ (1)

and denote by $P_{wac}(\tilde{A})$the completion of the locally convex space whose topology is generated by the seminorm in (1).

Quantum Solow Growth Model with Endogenous Delays

Solow growth model is defined by the O.D.E

$$\frac{dx}{dt} = sf(x(t)) - n_s\, x(t) (2)$$

where $x = C/L$ denotes the Capital-Labour ratio; $f(x)$ is the production per unit of labour, $s$is the saving rate ($0 < s < 1$), $n_s > 0$ is the rate of change of the labour supply $\frac{dL}{dt}/L$which was initially assumed to be exogenous by Solow.
The current rate of change of the labour supply is related to past fertility, and thus to past levels of wage, following a prescribed pattern of delay. There are two main alternatives: fixed delays and distributed delays. The former is better suited when there is no variability in the process of transmission of the past into the future; for instance: when we assume that all individuals are recruited in the labour force at the same fixed age. Conversely, when recruitment may occur at different ages, i.e. with different delays (for instance because the time needed to complete formal education is heterogeneous within the population), distributed delays appear more suitable. Quantum stochastic calculus was designed to describe the dynamics of quantum processes and we propose that we use it to study the non-commutative Solow growth model. The introduction of a distributed delay in the population term in (2) leads to the following quantum integro-differential equation

$$\frac{dx(t)}{dt}(\eta,\xi) = sf\big(x(t)\big)(\eta,\xi) - \left[\int_{-\infty}^{t} n_s\Big(f\big(x(\tau)\big)\Big)\, g(t-\tau)\,d\tau\right] x(t)(\eta,\xi) \quad (3)$$

for arbitrary$\eta,\xi \in D \underline{\otimes} E$where the term $n_s(f(x(\tau)))$, $\tau < t$ captures the past income-related fertility, and $g(t - \tau)$ is the corresponding delay kernel. Quantum integro-differential equation of this type in problem (3) are the generalization of the major economic applications known as vintage capital models (VCMs). The VCMs bring a new type of stability and optimization problems that involve the optimal control of an endogenous delay [19, and references therein]. On the other hand, the state of economic processes is often subject to shocks at certain instants, which may be caused by population changes, financial-structural and technological changes, which always implies impulses.

Evolution of quantum financial models as prompted us to extend Solow growth model with impulsive effects to describe the evolutionary process of the systemssince delays and impulses can affect the dynamical behaviours of the system. It is further necessary to investigate both the delay and the impulsive effects on the stability of the economic model.

For each pair$\eta,\xi \in D \underline{\otimes} E, t_0 \in I$we consider the following quantum impulsive Solow growth model with endogenous delay

$$\begin{cases} \frac{dx(t)}{dt}(\eta,\xi) = sf\big(x(t)\big)(\eta,\xi) - \left[\int_{-\infty}^{t} n_s\Big(f\big(x(\tau)\big)\Big)\, g(t-\tau)\,d\tau\right] x(t)(\eta,\xi), t \geq t_0, t \neq t_k, \\ \Delta x(t_k)(\eta,\xi) = x(t_k + 0) - x(tk) = J_k x(t_k), \qquad k = 1,2,\cdots\cdots\cdots \end{cases} \quad (4)$$

Where $x : N \subset \tilde{A} \to \tilde{A}$; $f: \tilde{A} \to \tilde{A}$, $g : \mathbb{R} \to \mathbb{R}_{+}$is the delay kernel function$0 < s < 1$, $n_s : \mathbb{R} \to \mathbb{R}$, $\mathrm{t_k} < \mathrm{t_{k+1}} < \cdots..,(k = 1,2,....)$are the moments of impulsive perturbations due to which capital/labour ratio changes from position $x(t_k)$ to $x(t_{k+0})$; $J_k$are constants which represent the magnitude of the impulsive effect at the moment $t_k$and $\lim_{k\to\infty} t_k = \infty$ System (4) is the quantumgeneralization of the Solow growth equation with endogenous delay [19].The case where $\triangle x(t_k)(\eta,\xi) < 0$corresponds to instantaneous reduction of the capital-labour ratio at timest_k; while the case
$\triangle x(t_k)(\eta,\xi) > 0$ describes heavy intensification of the capital- labour ratio.

Definitions

1. Let $x_0 \subset Pad\big(I,\tilde{A}\big)_{wac}$ such that $x(t)(\eta,\xi) = x(t; t_0 , x_0)$, $x_{\eta\xi} \in \tilde{A}$, for easy of notation$x(t)(\eta,\xi)$ is denoted by X be the solution of the system (4) satisfying the initial conditions
$$\begin{cases} \mathrm{x}_{\eta,\xi}(t;\ t_0,x_0) = x_0(t - t_0)(\eta,\xi), t \in (-\infty,t_0), \\ \mathrm{x}_{\eta,\xi}(t+0;\ t_0,x_0) = x_0(0)(\eta,\xi), \quad \eta,\xi \in \mathrm{D} \underline{\otimes} \mathrm{E} \end{cases} \cdots \cdots .(5)$$
and let$H^+ = H^+(t_0 , x_0)(\eta,\xi)$,the maximal interval in which the solution $x_{\eta\xi}(t; t_0 , x_0)$, is defined. While
$$||x_0||_{\eta,\xi} = \max_{t\in(-\infty,t_0]} ||x_0(t - t_0)||_{\eta,\xi}$$
is the norm of map $x_0 \subset Pad\big(I,\tilde{A}\big)_{wac}$.
2. The solution of system (4) satisfying (5) is said to be *equi-bounded* if for all $t_0 \in I$, $\alpha > 0$, there exist $\beta(t_0, \alpha) > 0$ such that for all $\phi_0 \in C(I, \tilde{A}) : ||\phi_0||_{\eta,\xi} < \alpha \quad \forall t \geq t_0; ||x(t; t_0, \phi_0||_{\eta,\xi} < \beta$
3. Is *Uniformly bounded*, if β in (2) is independent of $t_0 \in I$
4. Is *quasi-uniformly ultimately bounded* if there exist $\beta > 0$, $\forall \alpha > 0$ and also there exist $T = T(\alpha) > 0$ $\forall t_0 \in I$ such that$\phi_0 \in Pad(I, \tilde{A})_{wac}$: $||\phi_0||_{\eta,\xi} < \alpha \quad \forall t \geq t_0 + T$; $||x(t; t_0, \phi_0||_{\eta,\xi} < \beta$
5. Is *Uniformly ultimately bounded* if (3) & (4) hold
6. The solution of system (4) satisfying (5) is said to be *Stable* if for all $t_0 \in I$, $\epsilon > 0$, there exist$\delta = \delta(t_0, \epsilon) > 0$ such that for all
$\phi_0 \in C(I, \tilde{A}) : ||\phi_0||_{\eta,\xi} < \delta \ \forall t \geq t_0$; $||x(t; t_0, \phi_0||_{\eta,\xi} < \epsilon$
7. Is *Uniformly stable*, if δ in (6) is independent of $t_0 \in I$
8. Is *equi-attractive* if there exist $\lambda = \lambda(t_0) > 0$,$\forall \epsilon > 0$ and also there exist
$T = T(t_0, \epsilon) > 0$,$\forall t_0 \in I$ such that
$\phi_0 \in Pad(I, \tilde{A})_{wac}$: $||\phi_0||_{\eta,\xi} < \lambda$, $\forall t \geq t_0 + T$; $||x(t; t_0, \phi_0||_{\eta,\xi} < \epsilon$
9. Is *Uniformly attractive*, if λ and T in (8) is independent of $t_0 \in I$
10. Is *Uniformly asymptotically stable* if (7) & (9) hold.

Let us consider the following Assumptions

A_1 The delay kernel g : R → R_+ is continuous , and there exist a positive number ρ such that

$$\int_{-\infty}^{t} g(t-\tau)d\tau \le \rho < \infty, \forall t \in [t_0, \infty), t \ne t_0, k = 1, 2,$$

A_2 The map f is continuous on A˜, $f(x) > 0$ for $x > 0$, $f(0) = 0$ and there exist a positive continuous function $a(t)$ such that

$$\left(\frac{f\left(x_1(t)\right)(\eta,\xi)}{\left(x_1(t)\right)(\eta,\xi)} - \frac{f\left(x_2(t)\right)(\eta,\xi)}{\left(x_2(t)\right)(\eta,\xi)}\right)\frac{1}{x_1(t) - x_2(t)} \le -a(t)$$

for all $x_1, x_2 \in Pad(I,\tilde{A})_{wac}$ $x_1, x_2 \ne 0, x_1 \ne x_2$ for all $t \in I, t \ne t_k$

A_3 The map n_s is continuous on R, $n_s(f(x))(\eta, \xi) > 0$, for $n_s(f(0)) = 0$ and there exist a positive continuous function $b(t)$ such that

$\|n_s(f(x_1(t))) - n_s(f(x_2(t)))\|_{\eta,\xi} \le b(t)\|x_1(t) - x_2(t)\|_{\eta,\xi}$ for all $x_1, x_2 \in Pad(I,\tilde{A})_{wac}$ and $b(t)$ is non-increasing for $t \in I$,

$t \ne t_k$ $k = 1, 2, \ldots$

A_4 $t_0 < t_1 < \ldots.$ and $\lim_{k\to\infty} t_k = \infty$

A_5 Let $C^m < \infty$, $C^L > 0$, where $C^m = \max[C_i]$ and $C^L = \min[C_i]$ for $1 \le i \le n$

In the proof of our main theorems, we shall use the following results

Lemma 1

Let hypothesis $A_1 - A_4$ hold, and

$$\int_{-\infty}^{t} n_s(f(x(\tau)))g(t-\tau)d\tau$$

be continuous for all $t \ge t_0$. Then $H^+(t_0, x_0) = I$

Proof

If $\int_{-\infty}^{t} n_s(f(x(\tau)))g(t-\tau)d\tau$ is continuous for all $t \ge t_0$,then under the hypothesis $A_1 - A_4$ the system (4) and (5) has a unique solution $x(t)(\eta, \xi) = x(t; t_0, x_0)$ with $x_0 \in Pad(I, \text{A˜})_{wac}$. This means that solution $x(t)(\eta, \xi) = x(t; t_0, x_0)$ of problem (4) and (5) is defined on $[t_0, t_1] \cup (t_k, t_{k+1}]$, $k = 1, 2, \ldots$ From the hypothesis A_4, we conclude that it is continuable for $t \ge t_0$■

Lemma 2

Assume that:

i Condition of lemma I hold,

ii $x(t)(\eta, \xi) = x(t; t_0, x_0)$ is a solution of of problem (4) and (5) such that

$x(t)(\eta, \xi) = x(t - t_0)(\eta, \xi) \geq 0,$ $\sup x_0(s) < \infty,\ x_0(0) > 0$ (6)

iiiFor each $k = 1, 2, \ldots,$ $1 + J_k > 0$. Then $x(t)(\eta, \xi) > 0,$ $t \geq t_0$

Proof

By integrating problem (4) on the interval $[t_0, t_1]$ we have

$$x_i(t)(\eta,\xi) = x_1(t_0^+)\exp\left(\int_{t_0}^{t} F(\tau)d\tau\right)$$

Where $F_i(t)(\eta, \xi) = sf\big(x(t)\big)(\eta,\xi) - \left[\int_{-\infty}^{t} n_s\left(f\big(x(\tau)\big)\right) g(t-\tau)d\tau\right] x(t)(\eta,\xi), t \geq t_0\ , t \neq t_k$

Since in the interval $[t_0, t_1]$ there is no points of discontinuity of $x_i(t)$, from equation (6) it is obvious that $x_1(t) > 0$ for $t \in [t_0, t_1]$. Then $x_1(t) > 0$ we have from equation (5) that $x_i(t_1^+) =_1 x_i(t_1) + J_{i,1}x_i(t_1) + C_i, 1 \leq i \leq n$

from condition (iii) of lemma 2 and hypothesis A_5, it follows that

$x_i(t_1^+) = (1 + J_{i,1})x_i(t_1) + C_i > 0,$ $1 \leq i \leq n$

we continue integration of equation (4) on interval $[t_1, t_2]$ and we have

$$x_i(t)(\eta,\xi) = x_1(t_1^+)exp\left[\int_{t_0}^{t_1} F_i(\tau)\, d\tau\right]\ t \,\epsilon\, [\, t_1\, , t_2$$

from the above relation, it follows that $x_i(t)(\eta, \xi) > 0$ for $[t_1, t_2]$, $\eta, \xi \in D \otimes E$,

from the above relation, it follows that $x_i(t)(\eta, \xi) > 0$ for $t \in [t_1, t_2]$.

By similar arguments, we can obtain that

$$x_i(t)(\eta,\xi) = x_i(t_k^+)exp\left[\int_{t_k}^{t} F_i(\tau)\, d\tau\right]\ t \,\epsilon\, [\, t_k\, , t_{k+1}$$

$$for\ 1 \leq i \leq n, k = 1,2 \ldots, so\ x_i(t)(\eta,\xi) > 0\ , for\ t \geq t_0 \blacksquare$$

Theorem 1

Assume That

(i) Condition (i) and (ii) of lemma (2) hold

(ii)$-1 < J_k \leq 0$ for each $k = 1, 2, \ldots$

Then equation (4) is uniformly ultimately bounded.

Proof

From condition (i) and (ii) lemma (2). It follows that $t \in [t_0, t_1] \cup (t_k, t_{k+1}]$, $k = 1, 2, \ldots$, there exist a positive constants m^*i and$Mi* < \infty$ such that

$m^*i \le x(t) \le Mi*$

if we set $M^* = \max_k Mk*$, $k = 1, 2, \ldots.$, then by lemma (2) and condition(2) of this theorem, we have $0 < x(t_{k+0}) = (1 + J_k)x(t_k) \le x(t_k) \le M^*$ ■

Corollary 1

Let the condition of theorem I hold. Then there exist a positive constant m, $M < \infty$ such that

$m \le x(t) \le M,\ t \in I$

Let $\tilde{x}_0 \in Pad(I, \tilde{A})_{wac}$, and let $\tilde{x}(t) = \tilde{x}(t; t_0, \tilde{x}_0)$ be a solution of problem (4) for all $t \ge t_0$ with initial conditions

$\tilde{x}(t; t_0, \tilde{x}_0) = \tilde{x}_0(t - t_0),\ t \in I;\ \tilde{x}(t_0 + 0) = \tilde{x}_0(0)$

In the following, we shall suppose that

$x(t) = x_0(t - t_0) \ge 0, \quad \sup x_0(s) < \infty,\ x_0(0) > 0$

Theorem 2

(i) Condition of theorem 1 hold.

(ii)There exist a positive constant L such that

$$L_m + M_\rho \max_{t \in (-\infty, t]} b(\tau) \le msa(t), t \ge t_0, t \ne t_k, k = 1,2, \ldots$$

Then the solution $x(t)$ of problem (4) is uniformly asymptotically stable.

Proof

Define the Lyapunov function

$$V(t; x, \tilde{x}) = \left\| \frac{x}{\tilde{x}} \right\|_{\eta,\xi} \ldots\ldots\ldots (8)$$

By the mean value theorem, it follows that for any closed interval contained in $[t_0, t_1] \cup (t_k, t_{k+1})$, $k = 1, 2, \ldots$, we have

$\frac{1}{M}\ \|x(t) - \tilde{x}(t)\|_{\eta,\xi} \leq \|x(t) - \tilde{x}(t)\|_{\eta,\xi} \leq \frac{1}{m}\|x(t) - \tilde{x}(t)\|_{\eta,\xi}$ (9)

if $\|x_0(t) - \tilde{x}_0(t)\|_{\eta,\xi} < \delta < \infty$, then we obtain from inequality (9)

$V(t_0 + 0, x(t_0 + 0), \tilde{x}(t_0 + 0)) = \|x(t_0 + 0) - \tilde{x}(t_0 + 0)\|_{\eta,\xi}$

$\leq \frac{1}{m}\|x_0(t_0 + 0) - \tilde{x}_0(t_0 + 0)\|_{\eta,\xi} \leq \frac{1}{m}\|x_0 - \tilde{x}_0\|_{\eta,\xi} < \delta < \infty$ (10)

Considering the upper right-hand derivative $D^{+}V(t, x(t), \tilde{x}(t))$ of the map $V(t, x(t), \tilde{x}(t))$ with respect to problem (4). For $t \geq t_0$, $t \neq t_k$, $k = 1, 2, \ldots$, we derive the estimate

$$D_2^{+}\,V\left(t, x(t), \tilde{x}(t)\right) = \left(\frac{\frac{dx}{dt}(\eta,\xi)}{x(t)(\eta,\xi)} - \frac{\frac{d\breve{x}}{dt}(\eta,\xi)}{\breve{x}(t)(\eta,\xi)}\right) sgn\,(x(t) - \tilde{x}(t))$$

$$\leq s\left(\frac{f(x(t))(\eta,\xi)}{x(t)(\eta,\xi)} - \frac{f(\widetilde{x(t)})(\eta,\xi)}{\breve{x}(t)(\eta,\xi)}\right)\left(\frac{\|x(t) - \tilde{x}(t)\|_{(\eta,\xi)}}{x(t) - \tilde{x}(t)}\right)$$

$$+ \int_{-\infty}^{t} \left\| n_s\left(f\left(x(\tau)\right)\right) - n_s\left(f\left(\breve{x}(\tau)\right)\right)\right\|_{\eta,\xi}\ g(t-\tau)d\tau$$

$$\leq -sa(t)\left\|x(t) - \tilde{x}(t)\right\|\eta,\xi + \int_{-\infty}^{t} b(\tau)\,\|x(\tau) - \tilde{x}(\tau)\|\eta,\xi g(t - \tau)d\tau$$

From equation (9), using the Razumikhin condition

$V(\tau, x(\tau), \tilde{x}(\tau)) \leq V(t, x(t), \tilde{x}(t)),\ \tau \in (-\infty, t],\ t \geq t_0$

we have

$\leq \frac{1}{m}\|x(\tau) - \tilde{x}(t)\|_{\eta,\xi},\ \tau \in (-\infty, t],\ t \neq t_k,\ k = 1, 2, \ldots$(11)

From equation (11) and condition (2) of theorem (2), we obtain

$D^{+}V(t, x(t), \tilde{x}(t)) \leq -L\|x(t) - \tilde{x}(t)\|_{(\eta,\xi)} \leq -L_m V(t, x(t), \tilde{x}(t)),\ t \geq t_0,\ t \neq t_k,\ k = 1, 2, \ldots$ (12)

Also for $t = t_{,}\ k = 1, 2, \ldots$, we have

$$V\left(t_k + 0, x(t_k + 0), \tilde{x}(t_k + 0)\right) = \left\|\frac{x(t_k + 0)}{\tilde{x}(t_k + 0)}\right\|_{\eta,\xi}$$

$$\left\|\frac{1 + J_k x(t_k)}{1 + J_k \tilde{x}(t_k + 0)}\right\|_{\eta,\xi} = V\left(t_1, x(t_1), \tilde{x}(t_1)\right)\ \ldots\ldots (13)$$

This shows that the solution $x(t)$ of problem(4) satisfying (5) is uniformly asymptotically stable ■

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rom equation (12) and (13), we have

$V(t, x(t), \tilde{x}(t)) \leq V(t_0 + 0, x(t_0 + 0), \tilde{x}(t_0 + 0)) exp[-Lm(t - t_0]$ (14) Then from (14),(9) and (10), we deduce the inequality

1

$V(t, x(t), \tilde{x}(t)) \leq m\|x_0 - \tilde{x}_0\|_{(\eta,\xi)} exp[-Lm(t - t_0] t \geq t_0$

Theorem 1

$\tilde{A}$